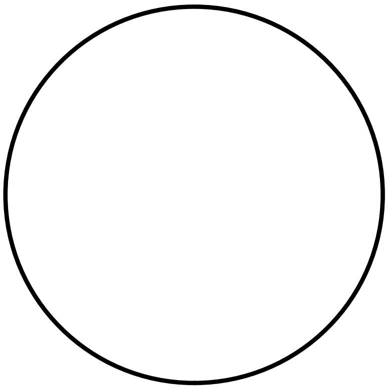
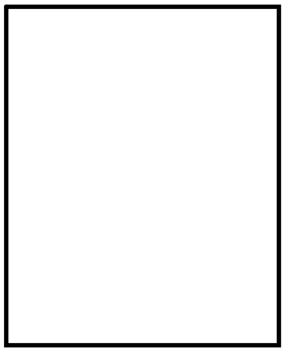
6.1: The Set of Rational Numbers

Definition: The rational numbers are all numbers of the form $\frac{a}{b}$, where *a* and *b* are integers with $b \neq 0$. We call *a* the numerator and *b* the denominator. We usually refer to these numbers in elementary school as fractions.

Example: Draw a figure to represent $\frac{1}{2}$ and $\frac{1}{3}$.





Example: Draw the points $\frac{-3}{2}$, 0, $\frac{3}{4}$, 2, $\frac{-7}{4}$, and $\frac{1}{2}$ on a number line.

6.1 Notes

Definition: In the fraction $\frac{a}{b}$, if $|a| < |b|$, we call it a proper fraction. If $|a| \geq |b|$, we call it an improper fraction.

Example: List some proper and improper fractions.

Proper: Improper:

Question: Is every integer a rational number?

Definition: Two fractions that represent the same rational number are known as equivalent fractions.

Example: Find fractions that are equivalent to $\frac{1}{2}$ by folding paper.

6.1 Notes

Fundamental Law of Fractions: If $\frac{a}{b}$ is any fraction and n is a nonzero integer, then $\frac{a}{b} = \frac{an}{bn}$.

Example: Show that $\frac{-7}{2} = \frac{7}{-2}$.

Example: Find a value for x such that $\frac{3}{12} = \frac{x}{72}$.

Definition: A rational number $\frac{a}{b}$ is said to be in <u>simplest form</u> if $b > 0$ and $\gcd(a,b) = 1$.

Example: Simplify the fraction $\frac{45}{300}$ by using the GCD.

6.1 Notes

Equality of Fractions: Show that $\frac{10}{16} = \frac{15}{24}$.

Theorem: Two fractions $\frac{a}{b}$ and $\frac{c}{d}$ are equivalent if and only if $ad = bc$. That is, we can cross multiply to check these.

Proof:

6.1 Notes

Theorem: If a, b, and c are integers with $b > 0$, then $\frac{a}{b} > \frac{c}{b}$ if and only if $a > c$.

Example: Show that $\frac{9}{12} > \frac{6}{9}$.

Theorem: If a, b, c and d are integers with $b, d > 0$, then $\frac{a}{b} > \frac{c}{d}$ if and only if $ad > bc$.

Proof: